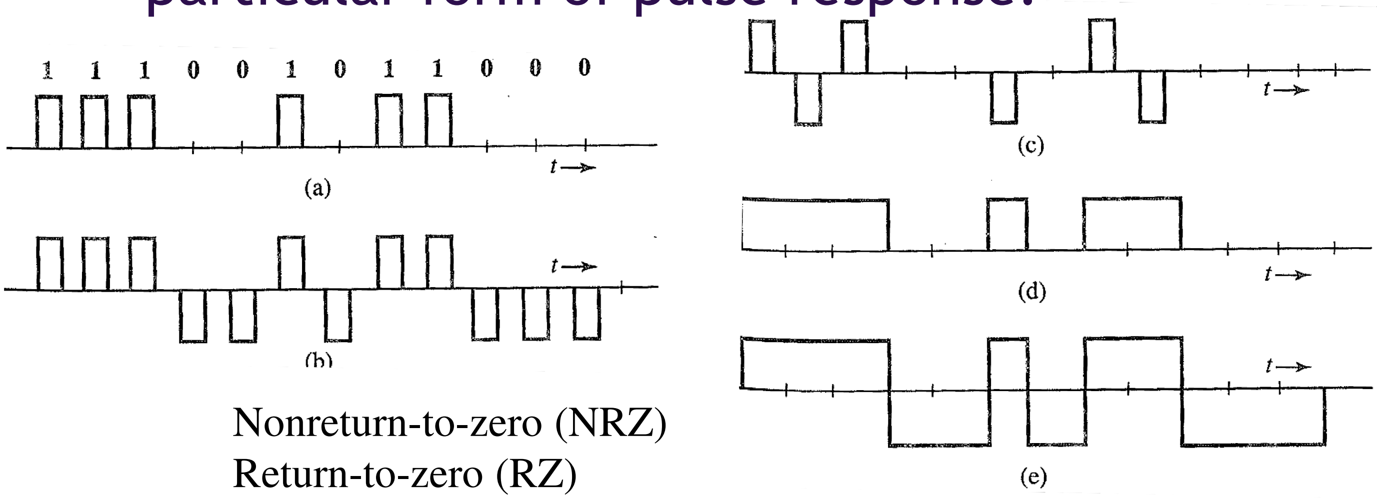
Digital Transmission
(Line Coding)

Pulse Transmission

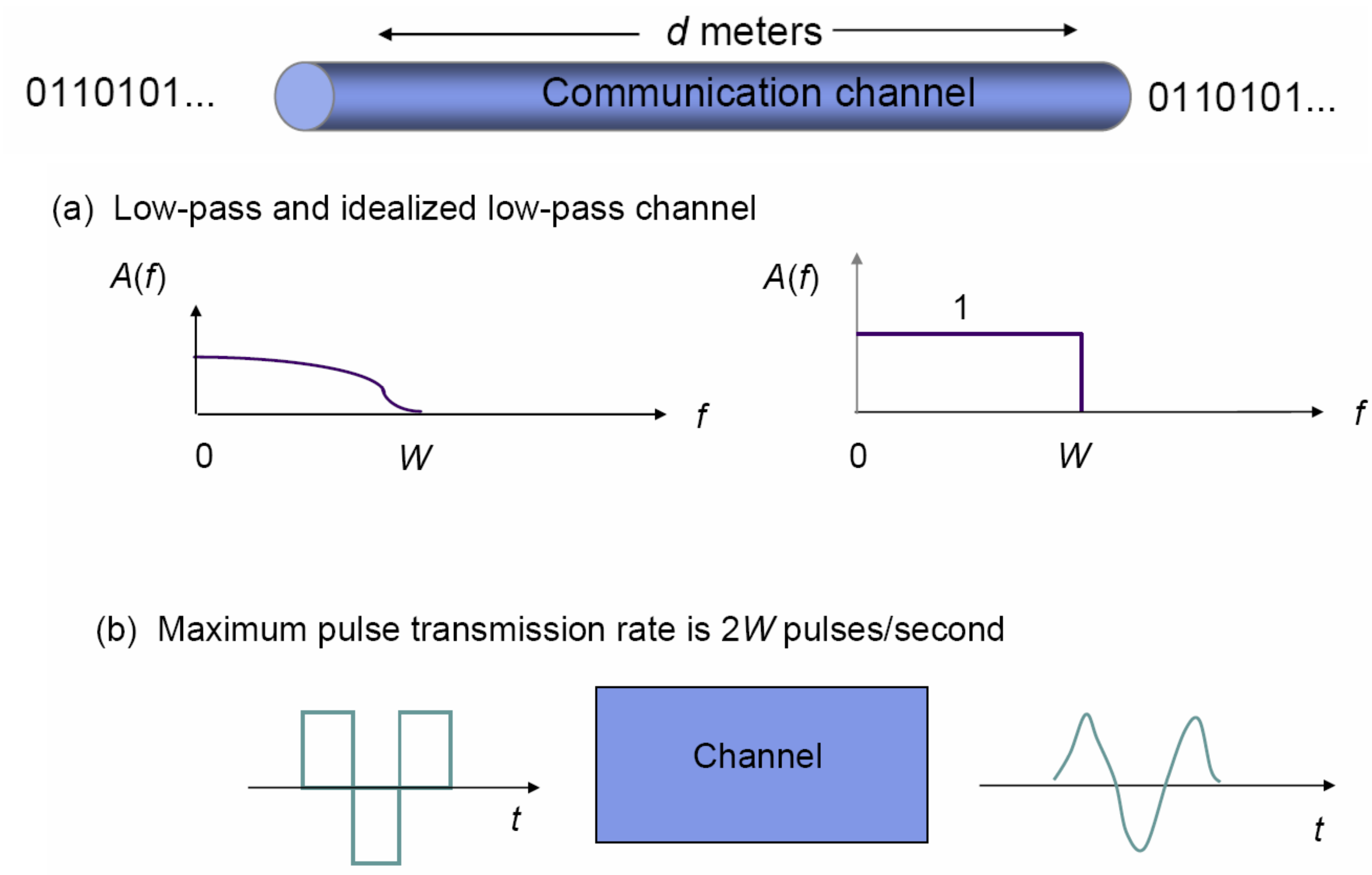
- Source → Multiplexer → Line Coder
- Line Coding: Output of the multiplexer (TDM) is coded into electrical pulses or waveforms for the purpose of transmission over the channel (baseband transmission)
- Many possible ways, the simplest line code on-off
- All digital transmission systems are design around some particular form of pulse response.



Nonreturn-to-zero (NRZ)
Return-to-zero (RZ)

(a) On-off (RZ)
(b) Polar (RZ)
(c) Bipolar (RZ)
(d) On-Off (NRZ)
(e) Polar (NRZ)

Pulse Transmission over a Channel



Desirable Properties for Line Codes

- Transmission Bandwidth: as small as possible
- Power Efficiency: As small as possible for given BW and probability of error
- Error Detection and Correction capability: Ex: Bipolar
- Favorable power spectral density: dc=0
- Adequate timing content: Extract timing from pulses
- Transparency: Prevent long strings of 0s or 1s

Review: Energy and Power Signals

- An energy signal $x(t)$ has $0 < E < \infty$ for average energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

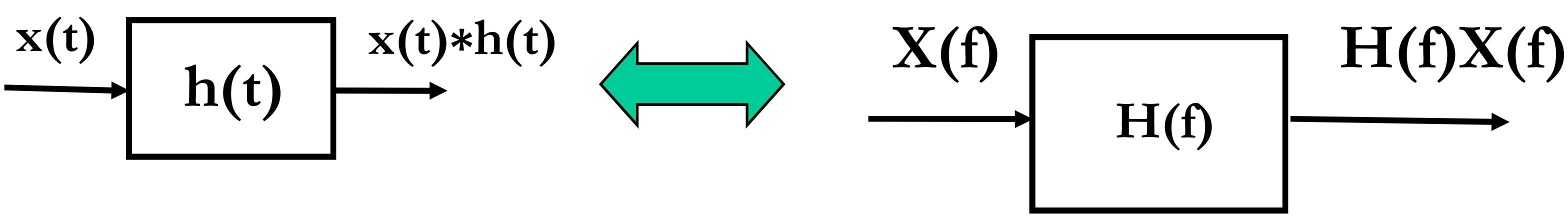
- A power signal $x(t)$ has $0 < P < \infty$ for average power

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

- Can think of average power as average energy/time.
- An energy signal has zero average power.
- A power signal has infinite average energy.
- Power signals are generally not integrable so don't necessarily have a Fourier transform.
- We use power spectral density to characterize power signals that don't have a Fourier transform.

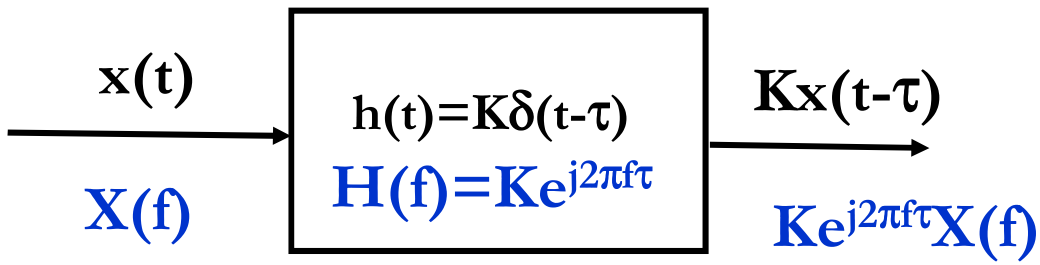
Review: Time-Invariant Systems

- Linear Time-Invariant Systems
- System Impulse Response: h(t)
- Filtering as Convolution in Time
- Frequency Response: $H(f)=|H(f)|e^{j\angle H(f)}$

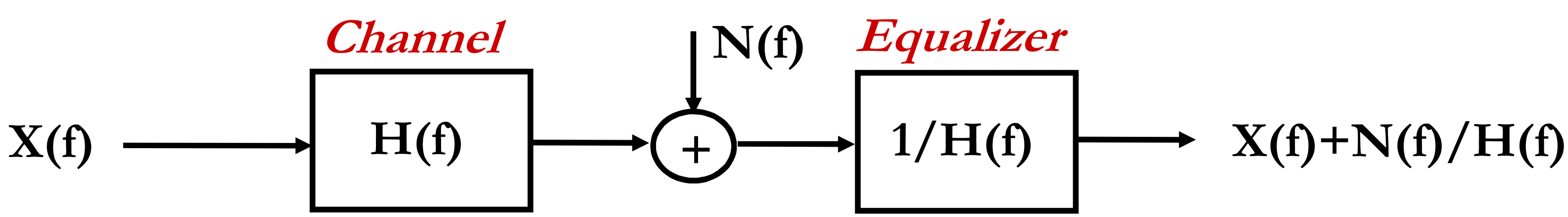


Review: Distortion

- Distortionless Transmission
  - Output equals input except for amplitude scaling and/or delay

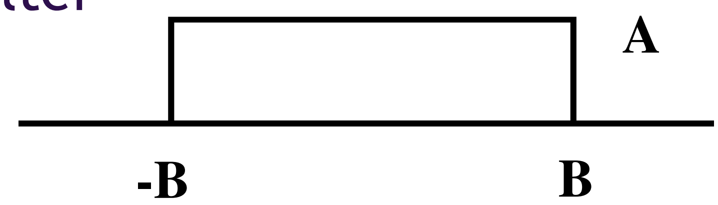


- Simple equalizers invert channel distortion
  - Can enhance noise power



Review: Ideal Filters

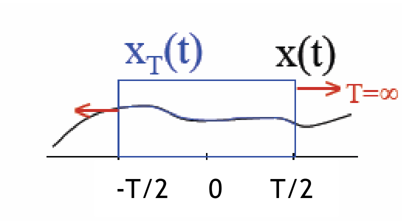
- Low Pass Filter



- Band Pass Filter

Power Spectral Density

- Power signals (P=Energy/t)
- Distribution of signal power over frequency



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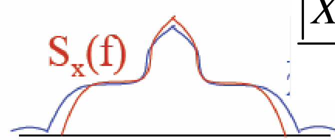
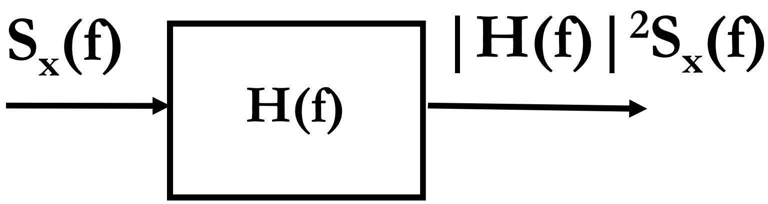


$$\frac{|X_T(w)|^2}{T}$$

$$S_x(w) = \lim_{T \to \infty} \frac{|X_T(w)|^2}{T}$$

- Useful for filter analysis



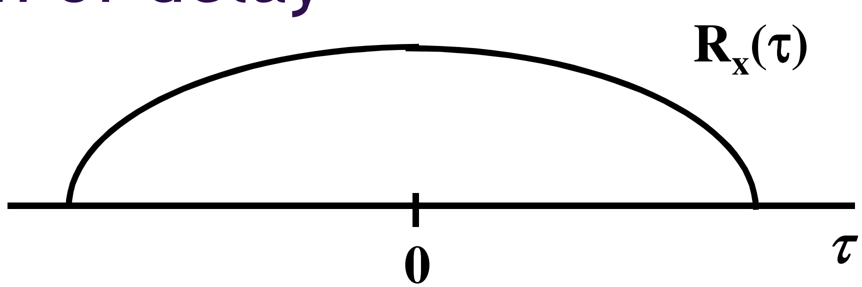
For $S_x(f)$ bandlimited [–B,B], B<<f_c

Definition: Autocorrelation

- Defined for real signals as $R_x(\tau)=x(\tau)*x(-\tau)$

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t-\tau)dt$$

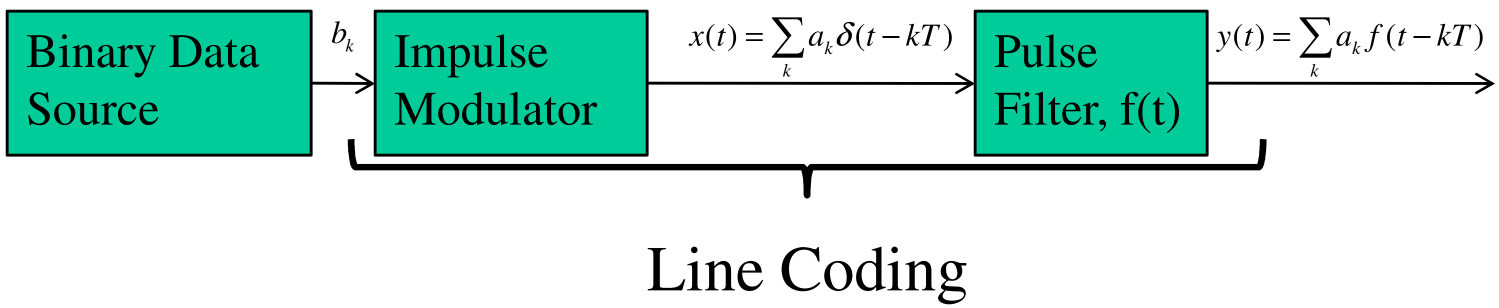
- Measures similarity of a signal with itself as a function of delay



- Useful for synchronization: $|R_x(\tau)| \leq R_x(\tau)$
- PSD and autocorrelation FT pairs: $R_x(\tau) \Leftrightarrow S_x(f)$

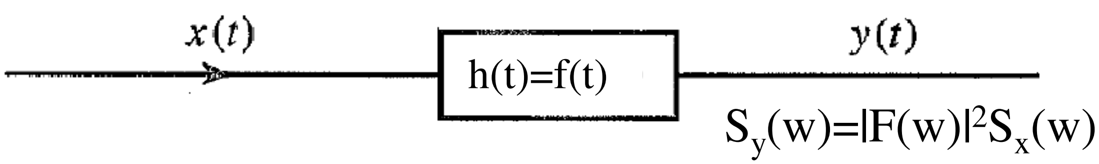
Bandwidth Usage of Line Codes

- Line codes are used for digital base-band modulation in data communication applications,
  - Digital data stream is encoded into a sequence of pulses for transmission through a base-band analog channel.
- The spectral properties of the line codes.
- We need a procedure for finding the PSD of line codes



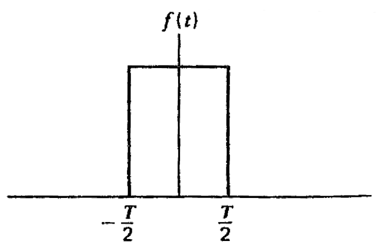
PSD Estimation

- We consider line coding pulses as a pulse train constructed from a basic pulse f(t) repeating at intervals of T with relative strength a_k for the pulse starting at t=kT such that the k^{th} pulse in this pulse train y(t) is $a_k f(t-kT)$.
  - For instance, the on-off, polar, and bipolar line codes are all special cases of this pulse train y(t), where a(k) takes on values 0,1, or -1 randomly subject to some constraints.
- We can analyze the various lines codes from the knowledge of the PSD of y(t)
- Simplify the PSD derivation by considering x(t) that uses a unit impulse response for the basic pulse of f(t).



Power Spectral Density

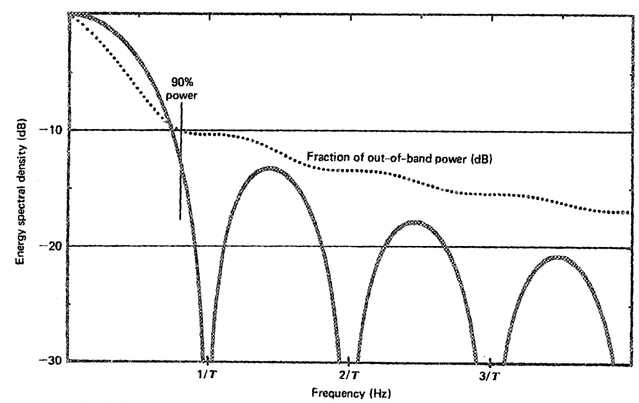
- PSD is the Fourier Transform of autocorrelation
- Rectangular pulse and its spectrum



$$f(t) = \begin{cases} 1 & |t| \le \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$F(w) = (T)\frac{\sin(wT/2)}{wT/2}$$

where w = radian frequency $2\pi f$,
T = duation of a signal interval.



PSD Derivation

- We now need to derive the time autocorrelation of a power signal x(t)

$$R_x(\tau) = \lim_{T_p \to \infty} \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} x(t)x(t+\tau)dt$$

- Since x(t) consists of impulses, $R_x(\tau)$ is found by

$$R_x(\tau) = \frac{1}{T} \sum_{n=-\infty}^{\infty} R_n \delta(\tau - nT)$$

  - where $$R_n = \lim_{N \to \infty} \frac{1}{N} \sum_k a_k a_{k+n}$$

- Recognizing $R_n = R_{-n}$ for real signals, we have

$$S_x(w) = \frac{1}{T} \left(R_0 + 2 \sum_{n=1}^{\infty} R_n \cos nwT \right)$$

PSD Derivation

- Since the pulse filter has the spectrum of F(w) $\leftrightarrow$ f(t), we have

$$\begin{aligned} S_y(w) &= |F(w)|^2 S_x(w) \\ &= |F(w)|^2 \left(\sum_{n=-\infty}^{\infty} R_n e^{-jnwT_b} \right) \\ &= \frac{|F(w)|^2}{T} \left(R_0 + 2 \sum_{n=1}^{\infty} R_n \cos nwT \right) \end{aligned}$$

- Now, we can use this to find the PSD of various line codes.

PSD of Polar Signaling

- In polar signaling,
  - binary "1" is transmitted by a pulse f(t)
  - Binary "0" is transmitted by a pulse –f(t)
- In this case, a_k is equally likely to be 1 or -1 and a_k^2 is always 1.

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2 = \lim_{N\to\infty} \frac{1}{N}(N) = 1$$

  - There are N pulses and $a_k^2=1$ for each one.
  - The summation on the right-hand side of the above equation is N.
- Moreover, both a_k and a_{k+1} are either 1 or -1. So, $a_k a_{k+1}$ is either 1 or -1.
  - They are equally likely to be 1 or -1 on the average, out of N terms the product $a_k a_{k+1}$ is equal to 1 for N/2 terms and is equal to -1 for the remaining N/2 terms.

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right] = 0$$

$$S_y(w) = \frac{|F(w)|^2}{T} R_0 = \frac{|F(w)|^2}{T}$$

$$R_n = 0 \qquad n \geq 1$$

$$S_y(w) = \frac{T}{2}\text{sinc}^2\left(\frac{wT}{2}\right)$$

Bipolar Signaling

- Bipolar signaling is used in PCM these days.
  - A "0" is transmitted by no pulse
  - A "1" is transmitted by a pulse f(t) or –f(t), depending on whether the previous "1" was transmitted by –f(t) or f(t)
- With consecutive pulses alternating, we can avoid the dc wander and thus cause a dc null in the PSD. Bipolar signaling actually uses three symbols [f(t),0,-f(t)], and hence, it is in reality ternary rather than binary signaling.
- To calculate the PSD, we have

$$R_n = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+n} \qquad R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2$$

PSD of Bipolar Signaling

- On the average, half of the a_k's are 0, and the remaining half are either 1 or -1, with $a_k^2=1$. Therefore,

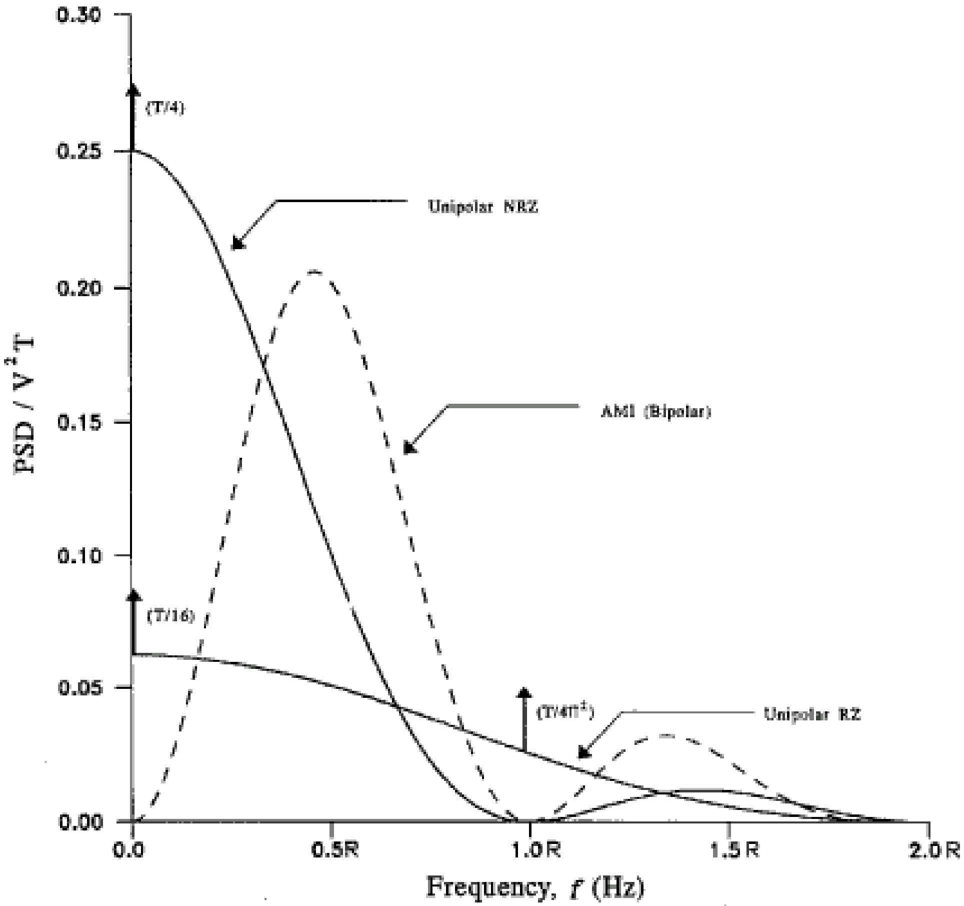
$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(\pm 1)^2 + \frac{N}{2}(0)^2\right] = \frac{1}{2}$$

- To compute R_1, we consider the pulse strength product $a_k a_{k+1}$.
  - Four possible equally likely sequences of two bits:11,10,01,00.
  - Since bit 0 encoded by no pulse (a_k=0), the product, $a_k a_{k+1}$=0 for the last three of these sequences. This means that, on the average, 3N/4 combinations have $a_k a_{k+1}$=0 and only N/4 combinations have non zero $a_k a_{k+1}$. Because of the bipolar rule, the bit sequence 11 can only be encoded by two consecutive pulse of opposite polarities. This means the product $a_k a_{k+1}$= -1 for the N/4 combinations.

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{4}(-1) + \frac{N}{4}(0)\right] = -\frac{1}{4}$$

PSD of Lines Codes

- PSD of lines codes

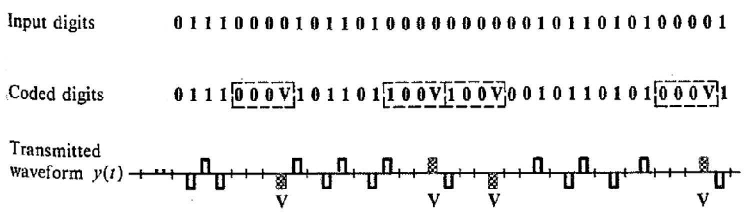


Binary N-zero Substitution (BNZS)

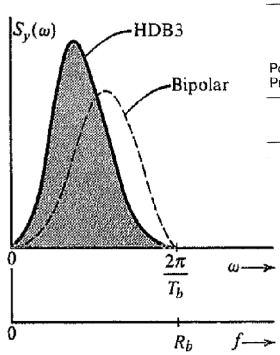
- Bipolar signaling has several advantages: (1) its spectrum has a dc null. (2) its bandwidth is not excessive. (3) it has single-error-detection capability. This is a due to the fact that if a single detection error is made, it will violate the alternating pulse rule.
- Disadvantages of bipolar signaling: it requires twice as much power (3 dB) as a polar signal. It is not transparent, i.e, we need a minimum density of 1's in the source to maintain timing at the regenerative repeaters. Low density of pulses increases timing jitter.
- Solution: Binary N-zero substitution (BNZS) augments a basic bipolar code by replacing all trings of N 0's with a special N-length code containing several pulses that purposely produce bipolar violations.

BNZS Line Codes

- High Density Bipolar (HDB) coding is an example of BNZS coding format. It is used in E1 primary digital signal.
  - HDB coding replaces strings of four 0's with sequences containing a bipolar violation in the last bit position. Since this coding format precludes strings of 0's greater than three, it is refereed to as HDB3 coding.



(a)



Polarity of Preceding Pulse	Number of Bipolar Pulses (1's) Since Last Substitution: Odd	Even
–	000–	+00+
+	000+	–00–

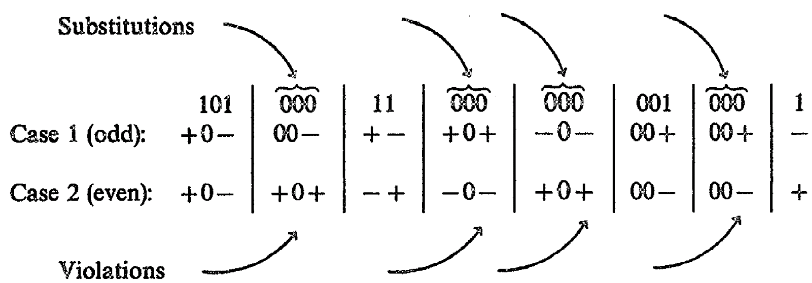
000V and B00V, where B=1 conforms to the bipolar rule and V=1 violates the bipolar rule. The choice of sequence 000V or B00V is made in such a way that consecutive V pulses alternate signs in order to maintain the dc null in PSD.

- B00V is used when there is an even number of 1's following the last special sequence
- 000V is used where there is an odd number of 1's following the last sequence.

B3ZS Line Code

- B3ZS Algorithm (as used DS-3 signal interface): Each string of three 0's in the source data is encoded with either 00v or B0V.

	Number of Bipolar Pulses (1's) Since Last Substitution	
Polarity of Preceding Pulse	Odd	Even
–	00–	+0+
+	00+	–0–



B8ZS Signaling

- B8ZS line code is used for T1 (DS1 signals) lines. It replaces any string of eight zeros in length with a sequence of 1's and 0's containing two bipolar violations. There are two bipolar violations in every substitution.

Polarity of Preceding Pulse	Substitution
−	000−+0+−
+	000+−0−+

- Similarly, in B6ZS code used in DS2 signals, a string of six zeros is replaced with 0VB0VB.

Polarity of Pulse Immediately Preceding Six 0's to be Substituted	Substitution
−	0 − + 0 + −
+	0 + − 0 − +

Example:

	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
+	−	(0	−	+	0	+	−)	+	0	−	+	(0	+	−	0	−	+)	(0	+	−	0	−	+)	0	0	0	−
−	+	(0	+	−	0	−	+)	−	0	+	−	(0	−	+	0	+	−)	(0	−	+	0	+	−)	0	0	0	+

Differential Encoding

- One limitation of polar signaling is that the signal for a 1 is exactly the negative of a signal for a 0. On many transmissions, it may be impossible to determine the exact polarity or an absolute phase reference.
  - The decoder may decode all 1's as 0's or vice versa.
- Common remedy for the phase ambiguity is to use differential encoding that encodes a 1 as a chance of states and encodes a 0 as no change in state. In this way, we do not need absolute phase reference.

Differential Encoding

- The differentially encoded sequence $\{d_k\}$ is generated from the input binary sequence $\{m_k\}$ by complementing the modulo-2 sum of m_k and d_{k-1}. The effect is leave the symbol d_k unchanged from the previous symbol if the incoming binary symbol m_k is 1, and to toggle d_k if m_k is 0.

$$d_k = \overline{m_k \oplus d_{k-1}}$$

$\{m_k\}$		1	0	0	1	0	1	1	0
$\{d_{k-1}\}$		1	1	0	1	1	0	0	0
$\{d_k\}$	1	1	0	1	1	0	0	0	1

- The decoder merely detects the state of each signal interval and compares it to the state of the previous signal.
  - If changed occurred, a 1 is decoded. Otherwise, a 0 is determined.

$$m_k = \overline{d_k} \oplus d_{k-1}$$

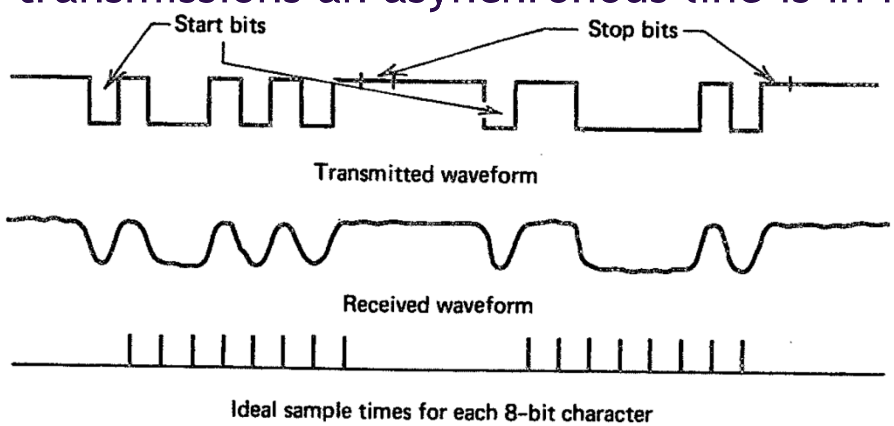
d_k	1	1	0	1	1	0	0	0	1
$\bar{d}_k$	0	0	1	0	0	1	1	1	0
d_{k-1}		1	1	0	1	1	0	0	0
m_k		1	0	0	1	0	1	1	0

Applications of Line Coding

- NRZ encoding: RS232 based protocols
- Manchester encoding: Ethernet networks
- Differential Manchester encoding: token-ring networks
- NRZ-Inverted encoding: Fiber Distributed Data Interface (FDDI)

Asynchronous vs Synchronous Transmission

- Asynchronous transmission: Separate transmissions of groups of bits or characters
  - The sample clock is reestablished for each reception
  - Between transmissions an asynchronous line is in idle state.

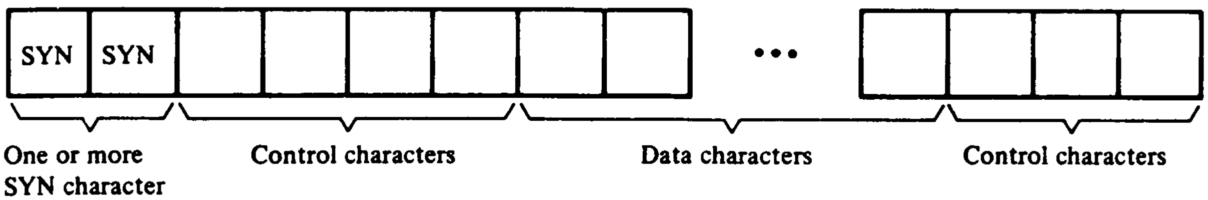


- Synchronous transmission: Digital signals are sent continuously at a constant rate
  - The sample clock is established and maintained throughout entire time.

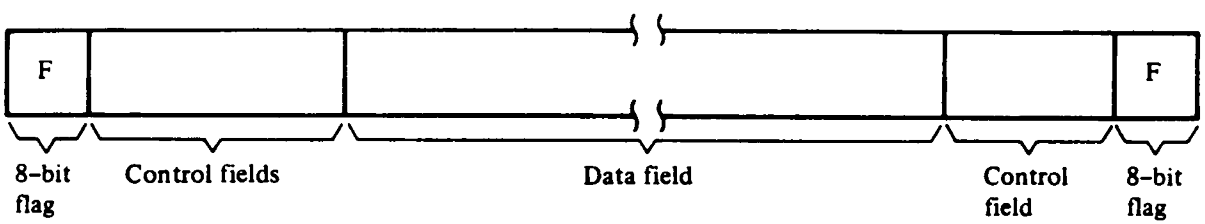
Synchronization Consideration

- Problem of **unvarying** signal
  - When a signal is unvarying, the receiver cannot determine the beginning and ending of each bit.
  - Take unipolar coding for example. **A long uninterrupted series of 1s or 0s can cause synchronization problem.**
- Problem of Using Timers
  - Whenever there is no signal change to indicate the start of the next bit in a sequence, the receiver has to rely on a timer. Given an expected bit rate of 1000 bps, if the receiver detects a positive voltage lasting 0.005 seconds, it reads one 1 per 0.001 seconds, or five 1s. However, five 1s can be stretched to 0.006 second, causing an extra 1 bits to be read by the receiver. That one extra bit in the data stream causes everything after it to be decoded erroneously.
- Problem of Having a Separate Clock Line
  - A solution developed to control the synchronization of unipolar transmission is to use a separate, parallel line that carries a clock pulse. But doubling the number of lines used for transmission increase the cost.

Synchronous Communication



(a) Character – oriented frame

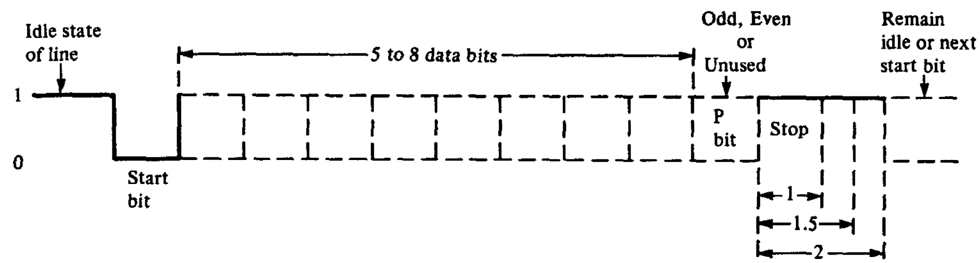


(b) Bit – oriented frame

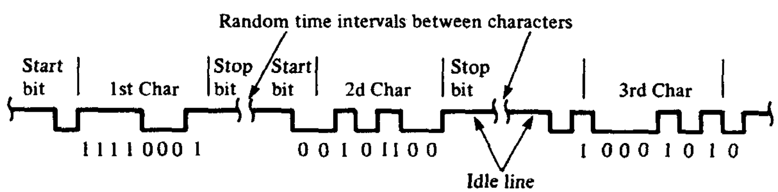
Asynchronous Transmission

- Bits are sent one character at a time. (A character is in general 8 bits in length)
- Timing or synchronization must only be maintained within each character. The receiver has the opportunity to resynchronize at the beginning of each new character.
- Start-stop technique
  - Idle state: When no character is being transmitted the line between transmitter and receiver is in an “idle” state. The definition of idle is by convention, but typically is equivalent to the signaling element for binary 1.
  - Start bit: The beginning of a character is signaled by a start bit with a value of binary 0.
  - Data bits
  - Stop bit: The last bit of the character is followed by a stop bit, which is a binary 1. A minimum length for the stop bit is specified and this is usually 1, 1.5 or 2 times the duration of an ordinary bit. No maximum value is specified, Since the stop bit is the same as the idle state.

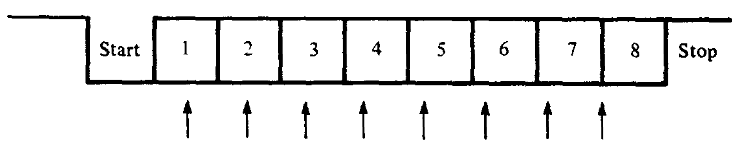
Asynchronous Communication



(a) Data character format

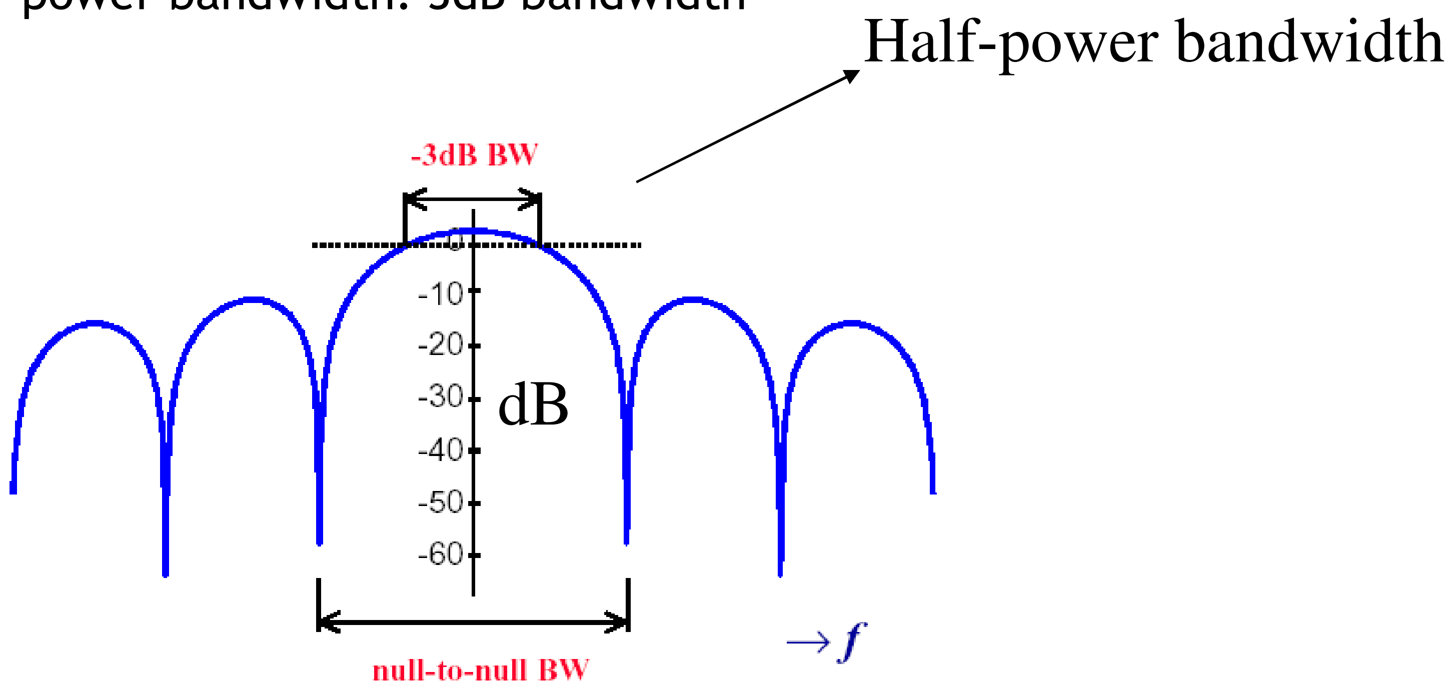


(b) 8 – bit asynchronous bit stream



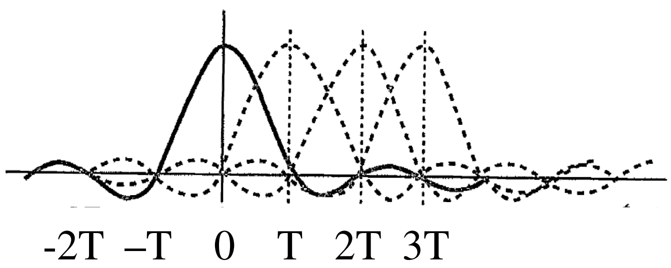
Bandwidth Definitions

- Measures of Bandwidth (BW):
  - 99% BW→ freq. range where 99% of power is
  - Absolute BW : Range of frequencies over a non-zero spectrum
  - Null-to-Null BW : Width of the main spectral lobe
  - Half-power bandwidth: 3dB bandwidth

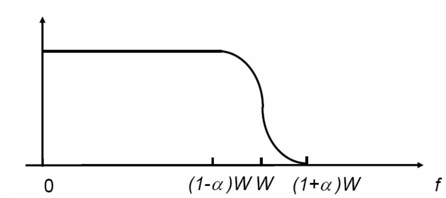


Pulse Shaping

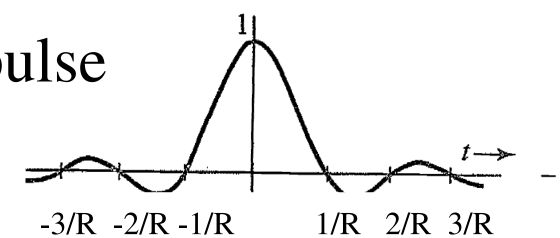
- Pulse shaping concerns with how to shape a pulse p(t) in order to achieve a desired $S_y(w)$.
- The PSD $S_y(w)$ is strongly and directly influenced by the pulse shape f(t) because $S_y(w)$ contains the term $|F(w)|^2$.
- Typical pulse response of a bandlimited channel



Nyquist Pulse or Raised-Cosine pulse



sinc pulse



Maximum Signaling Rate

- The percentage of total spectrum power is important measure
- A major result for digital transmissin pertains to the maximum rate at which pulses can be transmitted over a channel.
- If a channel has bandwidth W, then the narrowest pulse that can be transmitted over the channel has duration T=1/(2W) seconds.
- Thus, the maximum rate at which pulses can be transmitted through the channel is given by
  - $R_{max}=2\times W$ pulses/second.

Multilevel Signaling

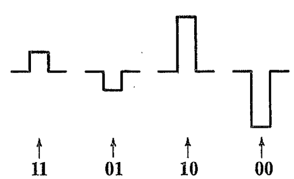
- Digital communications uses only a finite number of symbols for communication, the minimum being two (binary)
- Thus far, we have only considered the binary case.
- In some applications, the bandwidth is limited but higher data rates are desired, number of symbols (i.e., voltage levels) can be increased while maintaining the same signaling rate (baud rate).
- Multilevel signaling: The data rate R achieved by a multilevel system is given by

$$R = \log_2(L)\,(2W) = 2Wm$$

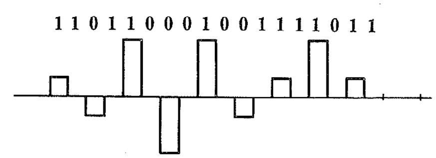
L is the number of levels

T is signaling interval.

Multilevel line codes



Multilevel transmission



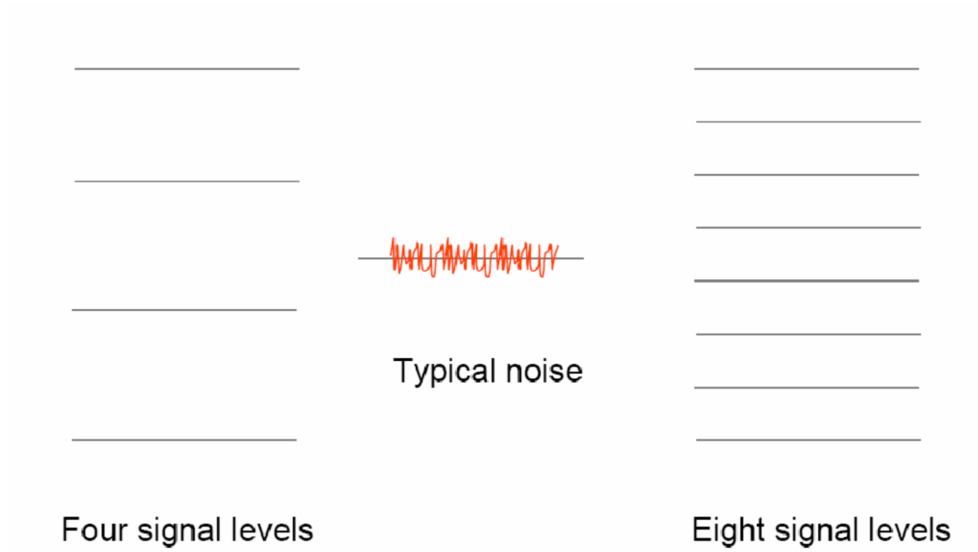
Multilevel Signaling and Channel Capacity

- Suppose we increase the number of levels while keeping the maximum signal levels $\pm$A fixed. Each increase in the number of signal levels requires a reduction in the spacing between levels. At some point, these reductions will imply significant increases in the probability of detection errors as the noise will be more likely to cause detection errors
- The channel capacity of a transmission system is the maximum rate at which bits can be transferred reliably. Shannon derived an expression for channel capacity of an ideal low-pass channel. He showed that reliable communication is not possible at rates above this capacity.

$$C = W \log_2(1 + \mathsf{SNR}) \quad \text{bits/second}$$

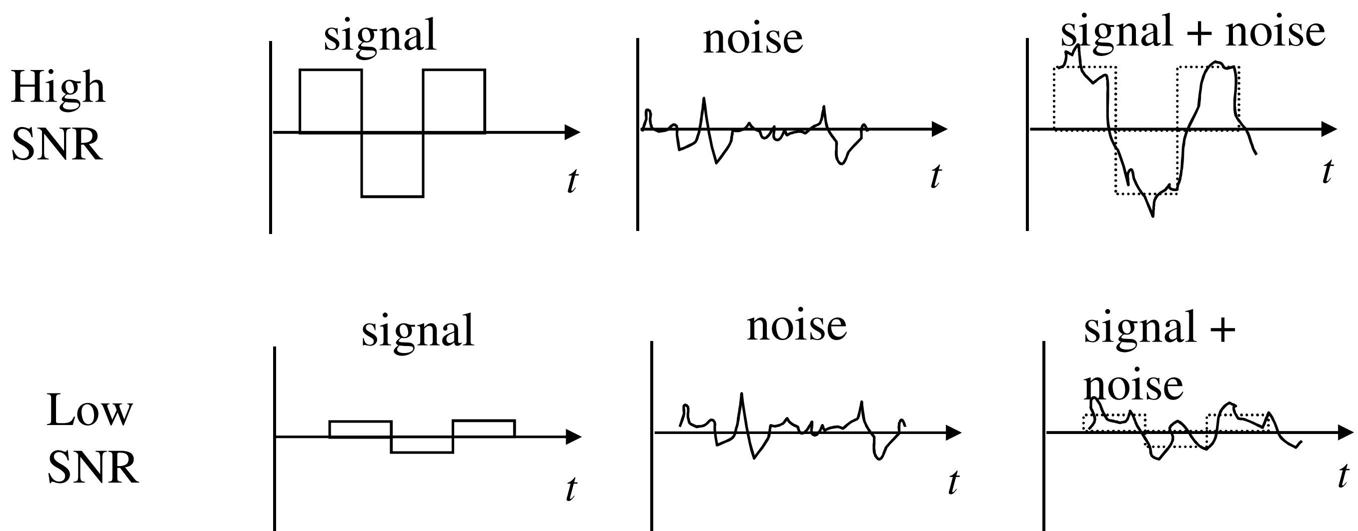
MultiLevel Signals and Noise

- Multilevel signaling and noise



Signal-to-Noise Ratio

- Definition of SNR



$$\text{SNR} = \frac{\text{Average Signal Power}}{\text{Average Noise Power}} = \frac{A^2}{\sigma^2}$$

$$\text{SNR (dB)} = 10 \log_{10} \text{SNR}$$

A=noise free sample voltage at the receiver

σ^2=the total noise power at the detector=(N_0)(NBW)

NBW=noise bandwidth

N_0=Power of white noise per Hertz

Error Performance

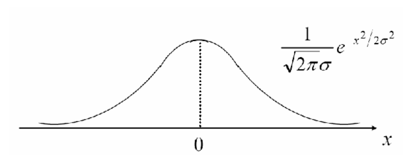
- Signal Detection: A decision of which signal was transmitted is made by comparing the measurement (at the appropriate time) to a threshold located halfway between these nominal voltages that represent “0” and “1”.
- Error performance depends on the nominal distance between the voltages and the amount of fluctuation in the measurements caused by noise.
- In absence of noise, the measurement of the positive pulse would be A and that of negative pulse would be –A. Because of noise, these samples would be $\mp A+n$ where n is the random noise amplitude.
- The error performance analysis in communication circuits is typically based on white Gaussian noise.

Error Probabilities

- We now compute the probability of error for a polar signal. The amplitude n of the noise is Gaussian distributed. It ranges from -∞ to ∞ according Gaussian PDF.
- When "0" is transmitted, the sample value of the received pulse is –A+n. If n>A, the sample value is positive and the digit will be detected wrongly as 1. If P(error|0) is the probability of error given that 0 is transmitted, then,

$$\begin{aligned} P(\text{error}|0) = Prob(n < -A) &= \frac{1}{\sigma\sqrt{2\pi}}\int_A^{\infty} e^{-n^2/2\sigma^2}dn \\ &= \frac{1}{\sqrt{2\pi}}\int_{A/\sigma}^{\infty} e^{-x^2/2}dx \\ &= Q\left(\frac{A}{\sigma}\right) \end{aligned}$$

$$Q(y) = \frac{1}{\sqrt{2\pi}}\int_y^{\infty} e^{-x^2/2}dx$$

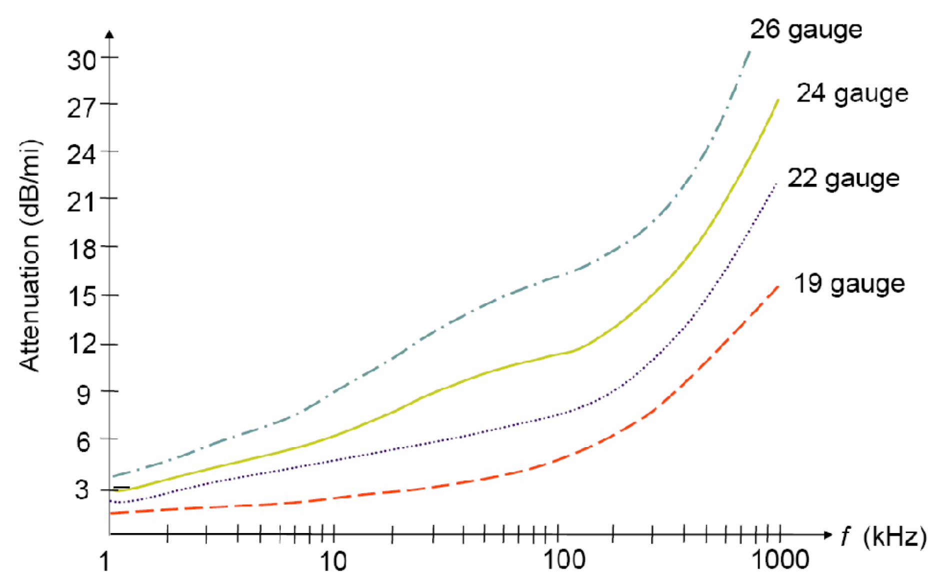


- Probability of error for a polar signal

$$P(\text{error}) = \frac{1}{2}[P(\text{error}|0) + P(\text{error}|1)] = Q\left(\frac{A}{\sigma}\right)$$

Twisted Pair

- A twisted pair consists of two wires that are twisted together to reduce the susceptibility to interference.



Gauge (diameter)

- The two-wire system is susceptible to crosstalk and noise since the multiple wires are bundled together.

Error Performance

- Polar Signaling

$$P(\text{error}) = Q\left(\frac{A}{\sigma}\right)$$

Power=A^2

- On-Off Signaling

$$P(\text{error}) = Q\left(\frac{A}{2\sigma}\right)$$

Power=$A^2/2$

- Bipolar Signaling

$$P(\text{error}) = 1.5Q\left(\frac{A}{2\sigma}\right)$$

Power=$A^2/2$

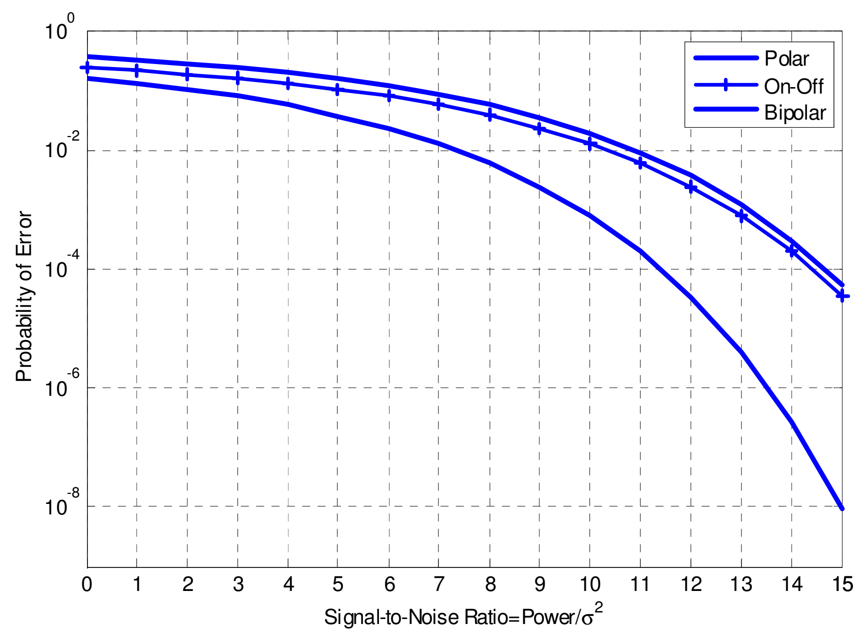
SNR=Power/σ^2

SNR=Energy/N_0

A→ Peak amplitude (Volts)

σ→ noise rms amplitude (Volts)

σ^2=total noise power



Performance Monitoring

- Redundancy Checks
  - **Parity Bits** are inserted into DS3 and DS4 signals for the purpose of monitoring the channel error rate.
  - The following equation relates the parity error rate (PER) to the channel probability of error or bit error rate (BER)

$$\text{PER} = \sum_{i=1}^{N} \binom{N}{i} p^i (1-p)^{N-i} \quad \text{(i odd)}$$

N=number of bits over which parity is generated
p=BER assuming independent errors

  - **Cyclic redundancy check (CRC) codes** are also incorporated into a number of transmission systems as a means of monitoring BERs and validating framing acquisition.
  - Examples of CRC use: Extended superframe (ESF) on T1 lines

$$\text{CRCER} = 1 - (1-p)^N$$

N=length of CRC field (including CRC bits)
p=BER assuming independent errors